MATHEMATICAL MODELING IN DESIGN OF TRANSPORT VEHICLES

Cârdei P.[1)], [2)]Atanasov At., Ciupercă R.[1)], Muraru V.[1)], Sfîru R.[1)]
[1)]INMA Bucharest; [2)]University of Rousse

Abstract: *The paper presents results related to comparing the behavior of different mathematical models for components of some classic transport vehicles. Agricultural trailers or semi-trailers high capacity have been more than two - three decades into the focus of researchers and designers for multiple purposes: increasing transport capacity and displacement speed, increasing work safety, optimizing specific qualities, etc. To address these problems it is necessary mathematical modeling of the real structures. This may be more or less complex. Complexity of the models depends on the desired results. It is important for economic reasons to work with models more simple and not refer directly to complex models involving a large number of hypotheses, which require numerous experimental verifications. The illustration of these issues is the aim of the article.*
Key words: *model, transport, vehicle, agricultural*

INTRODUCTION

Mathematical modeling in the structural analysis is an extremely large field, due to the complexity that can be achieved. A basic direction in which appears firstly the problem of models complexity is that of their geometric representation (geometric model).

Geometric model can be very simple, 1 or 2 dimensional, discretized with 1 or 2-dimensional finite element, but can also be very complex, 3-dimensional model discretized with three-dimensional finite elements. There are also, higher levels of complexity, hybrid in which physical entities modeled occur as ideal entities of different geometric dimensions.

The model geometry thus opens a wide field of mathematical modeling complexity, but it is not the only one. Another area which may increase the complexity of the mathematical models is the mathematical modeling of materials rheological properties, properties that give the behavior of materials under certain loading conditions: elastic, elasto–plastic and elasto-visco-plastic materials, linear or nonlinear, composites, etc..

Another source of mathematical models complication is the coupling of various phenomena: mechanical and thermal, fluid flowing and mechanical, mechanical and electro-magnetic, etc..

The main problem must to limit the complication of mathematical models, is that, any element that complicates a model requires one or more new hypotheses to be found and tested experimentally. A second problem that limits the complication of the model consists in its testing, which again requiring validation experiences. The third problem that prevents excessively complication of mathematical models is economy of intellectual and computational effort, problem that should not be neglected.

On the other hand, gradual complication of models must to give a measure of its profitability, ie the ratio between new results obtained toward the basic model and their price in intellectual, experimental and calculation effort. This paper will try to exemplify these aspects, regarding the new results that can be obtained on more complicated models, and, in the same time, highlighting necessary efforts to obtain them. The paper does not refer to experiments, because, mathematical models used do not involve assumptions which require such validations.

MATERIAL AND METHODS

The structure given as example in this article is the chassis of a trailer and its axle. It will illustrate gradual complexity of the model. It will start on an apparent logical direction, but which does not ensure passing through the shortest path. Obviously, elementary checking would be done starting with the simple element, the axle, for which there are classical, theoretical and test formulas, ie. the control of model functioning. The drawing provided by the designer appears in Fig. 1.

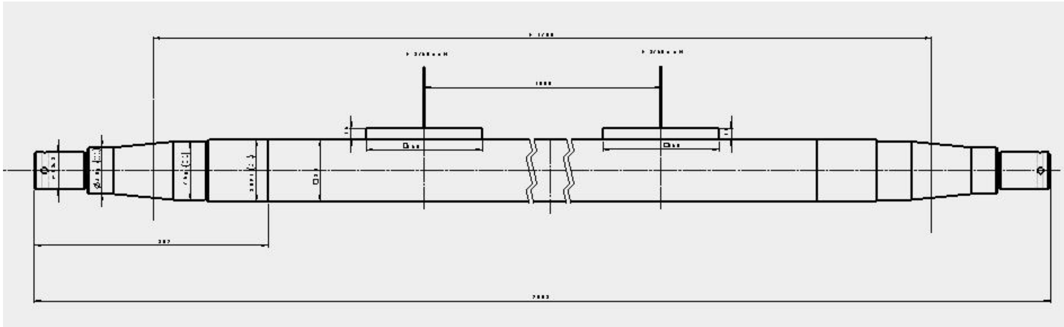
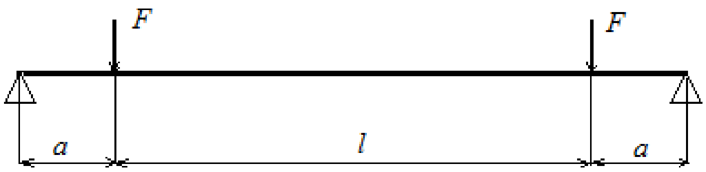


Fig. 1 - Technical design of the axle

M1- The axle elementary model as a bar with constant section

According to the designer's specifications, the simplest model is that, taken from specialized resistance books, which represents the axle as a straight beam (Bernoulli-Euler type model, [12] or [13]) of constant section with the opening equal with the distance between the supporting areas centers of the axle on bearings and loaded in the centers of the two symmetrical loading areas (pattern assigned as M1).

Such a model appears in many treaties of materials strength, eg [1], [3], [11], [15], [16], [17], [18] or by machine parts, [14]. The scheme and the model formulas are given in Fig. 2. The bar is articulated in two bearing points.



$$M_{\max} = Fa \quad (1)$$

$$u_{\max} = \frac{Fa\left(3l^2 - 4a^2\right)}{24EI} \quad (2)$$

Fig. 2 - Geometry, bearing and loading of the axle modeled within the M1 model

In formulas (1) and (2), M_{max} is the maximum moment, F is half of the axle total loading force that acting, a and l are geometric characteristics, E is the elasticity modulus of the material from which the axle is built, I is the geometric inertia moment of the bar cross section, that shapes the axle and U_{max} is the maximum relative displacement along the bar on its axis.

For the axle model in Fig. 2, the bar cross-section is constant with full square side of 80 mm, the material which the bar was built being OLC 45 (E=2.1·10^{11} Pa, the Poisson coefficient, ν = 0.3, mass density, ρ= 7850 kg/m^3).

M2, 1-dimensional finite element model

Nearest mathematical model with finite element toward the previous model that was solved purely analytically (with simple or elementary formulas), but which also respects the real geometric shape of the bearing areas, with bearings in one single point in each bearing area is given in Fig. 3.

The axle geometric model is an elementary one: straight bar (reduced to symmetry axis) with section of full square on the central area (square with side of 80 mm) and with circular section in the supporting areas (diameter of 75 mm). The areas at the ends, including two finite elements each, bordered by three nodes are the axle supporting areas on the bearings.

Discretization is done with 1-dimensional elements of BEAM3D type, found in the finite elements library of the structural analisys program used, [4]. The bearing is done by canceling translational displacements (u_x, u_y, u_z relative displacements) and rotational movement around its own axis of the axle (relative rotation, r_x). Free rotations around the O_y and O_z axis remain free.

Bearing described above is done in one single node, considering that, the bearing area is very limited and the system in which the bearing is mounted is bearing on elastic elements (wheels). The similar stifftening in the two other nodes in each of the two bearing areas leads to increases the tension in axle, which is not consistent with reality.

This bearing mode leads to results in agreement with literature specific in the field of materials resistance, [1]. The model with finite elements given in Fig. 3 and described above is denoted further with M2.

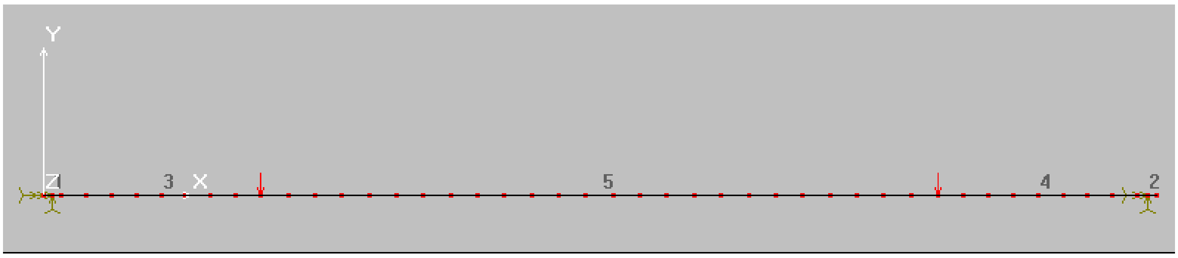


Fig. 3 - The elementary model with finite elements of the axle (geometry, bearing and loading, M2)

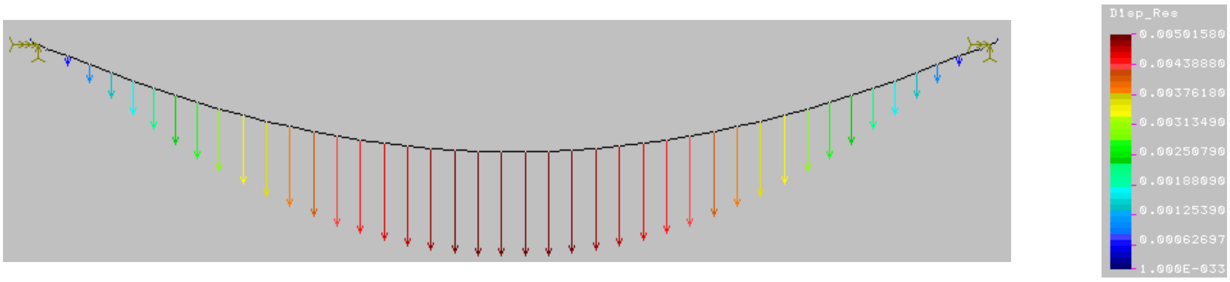


Fig. 4 - The map of relative displacement resultant in the M2 model of the axle, under the circumstances above, given vectorial on the deformed shape of the body. Values are given in m

Fig. 5 - The equivalent tension map in the M2 model of the axle, under the circumstances of loading specified above. Values are given in Pa (N/m^2)

Distribution of the relative displacement resultant and the equivalent tension in the M2 axle model is given through the map in graphic form in Fig. 4 and 5. The axle fundamental frequency is 71.76 Hz and the corresponding deformation is a bending one. Their own frequencies are: 287.06, 645.818, 794.115, 1148.01, 1794.06, 2381.24, 2584.35, 3517.98, 3965.05 Hz. The reaction force in each bearing points has the value of 37,500 points N, ie. half of the total load applied to the axle.

The problem of axle bearing generates a large number of possible models. The way which the axle is placed on the bearing, size and shape of the contact surfaces is unknown and changes over time, faster at the begining, according to loading. Changing the bearing conditions at the M2 mathematical model of finite elements leads to three different models which give results, that differ substantially from those of M1 and M2 models.

Geometry, discretization and load of M2 model variants will be denoted M21, M22 and M23. For each of these models, the bearing area is discretized into two elements and three nodes that limit them. The nodes at the ends of the two bars that limit the bar and the bearing area will be called external nodes. The nodes of the two areas situated towards the center of the axle, which are located at the axle section changing border will be called internal nodes of the bearing area.

The nodes located on the centers of the bearing zones will be called the central nodes. With these specifications can be defined, for differentiation, the models derived from M2

- The M2 model has the bearing only in the central nodes, $u_x=0$, only the left, $u_y=u_z=0$;
- The M21model, maintain the bearing of the M2 model, addition, in the extreme nodes: $u_y=u_z=0$
- The M22 model, maintain the bearing of the M2 model, addition, in the interior nodes: $u_y=u_z=0$;
- The M23 model, maintain the bearing of the M2 model, addition, in the extreme and interior nodes: $u_y=u_z=0$.

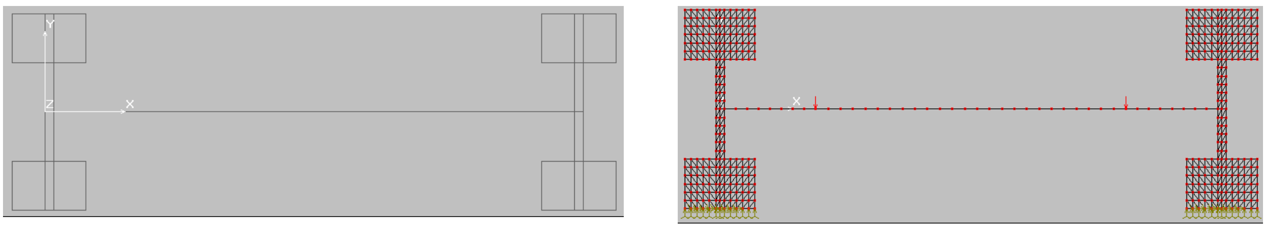
The main results of the M2 models and their derivatives appear in Table 1. As a result of many possible bearing assumptions, one can imagine many mathematical models with finite element, which have a higher level of complexity than a simple one-dimensional bar bearing on the ends.

One of these is that whose geometry, loading and bearing are given in Fig. 6. The model will be called M3. Rectangles on which the axle bearing directly (Fig. 6, a), shape rims of the wheels, and rectangles being in their extension shape tires, that represents the axle elastic bearing. These geometric entities are discretized with 2-dimensional finite elements by the SHELL3 type from the program with finite elements library [4].

Because in this model interests only the axle behavior, the rim and the tire models are not models with high fidelity to reality. The rims and the tires play in this model only the role of elastic bearings. Under these conditions, the results must be carefully interpreted.

The resultant relative displacement of the axle is calculated by the difference between its value on the entire structure at the middle of the axle and the value of the same field in the central node of the bearing. The value of this difference is about 4.98 mm. The maximum equivalent tension is located in the axle in the central part between the nodes in which the load is applied. Its value is 134.7 MPa.

The first ten own frequencies of the M3 model are: 0.0026, 12.181, 18.916, 19.432, 28.392, 79.147, 115.744, 116.681, 321.564, 335.12 Hz.



a) b)

Fig. 6 - Geometry (a), discretization, bearing and loading (b) of the M3 model

The chassis structural model, M4

The gradual complexity of the model continues with the chassis structural model, that bearings in the back on the axle and in the front part on the tractor coupling point. The basic structural model of the chassis, that will further be denoted with M4 is built exclusively with finite elements type BEAM3D, found in the finite elements library of the structural analysis program wherewith worked. In Fig. 9 *The structural model of chassis* appears, used for analysis in this calculation.

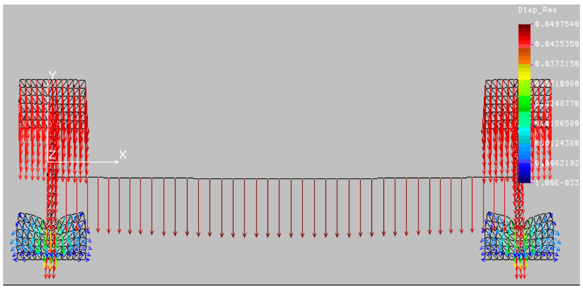


Fig. 7 - Distribution of the relative displacement field resultant in the M3 nodel. The values are given in m

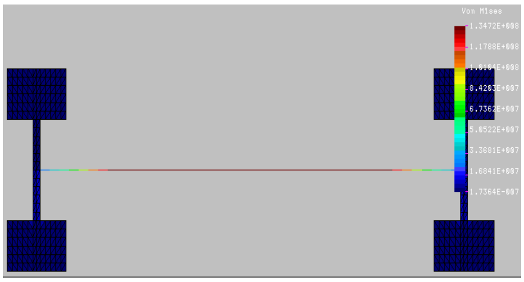


Fig. 8 - Equivalent tension distribution in the M3 model. The values are given in MPa

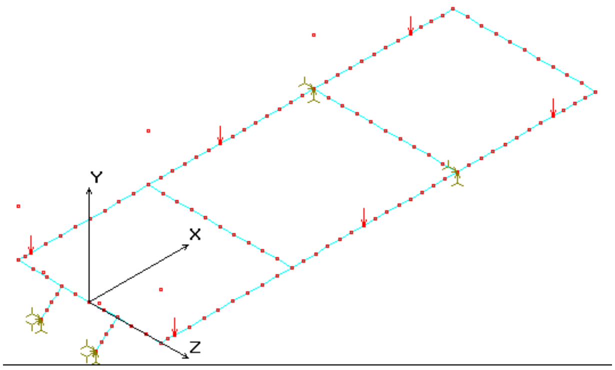


Fig. 9 - (M4) basic model with finite elements of the trailer chassis - geometry, discretization, bearings and loading

Were used for bearing the cancellation conditions of the three translational relative displacements in the front of the chassis (the one that connects to the tractor, $u_x=u_y=u_z=0$ and cancellation of vertical and lateral translations in the axle bearing points of the chassis ($u_y=u_z=0$).

The loading has been done with a total force of 7500 daN, in six points, according to the designer's indications, equally loaded, so 1250 daN in each loading point (red arrows in Fig. 9). According to the designer's indications, the two longerons were made of U 180x80x5 mm profile, as the traverse behind the trailer and the penultimate traverse from U 120x80x5 mm profile.

The first two traverses have been considered from steel with square pipe profile of 100x6 mm. The material is L42 steel for longerons (250 MPa yield limit), back and central traverse, S275 JR to ear hitch, OLT 35 front traverse (230 MPa yield limit). It took for all steel E = 2.1 $\cdot 10^{11}$ Pa, v = 0.3, respectively 7850 kg/m^3 density.

The analysis has been performed by the finite element method, obtaining the resultant relative displacement and the equivalent tension field in the structure, as is grafic represented in Fig. 10 and 11.

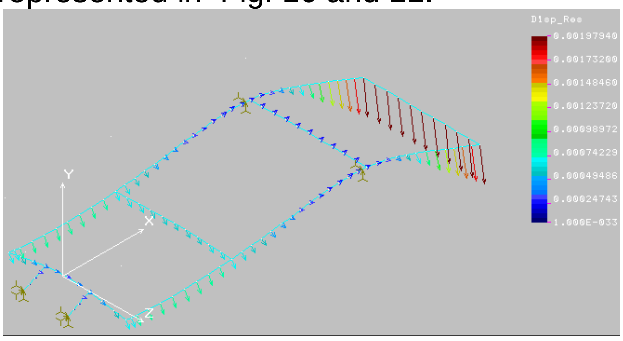


Fig. 10 - The relative displacement field distribution resulting in the structure, in the bearing and loading conditions specified above. Distribution is given on the structure deformed shape. The values are given in m

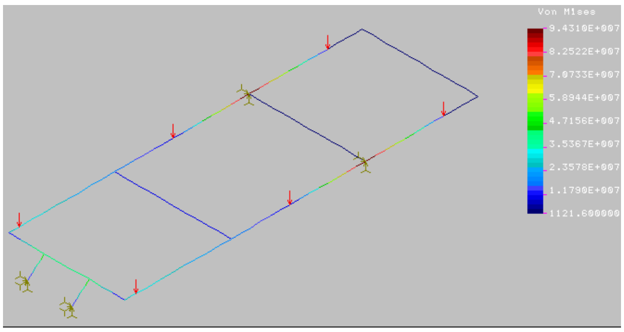


Fig. 11 - The equivalent tension field in the trailer chassis structure on the undeformed shape. The values are given in Pa (N/m2)

It is noted that, the arrow in the structure has a maximum value of about 2 mm (1.9794 mm), in the loading and bearing conditions specified, a value very small compared with the structure scale. The maximum tension in the structure components built from L42 steel is 94.31 MPa and the structure components made of OLT 35 is about 42 MPa.

The own fundamental frequency of the chassis, thus bearing is set to 50.98 Hz and the structure deformed shape is grafic represented in Fig. 12.

In fig. 13 are given the reactions in the chassis bearing points, the values being by 26465.7 N, on each of the bearing points on the axle and by 11053.9 N on each of the front bearing points from the hitch. Their sum exceeds the loading value with a force of 38 N, which is due to discretization calculation errors, but this error is only 0.05% of the total load.

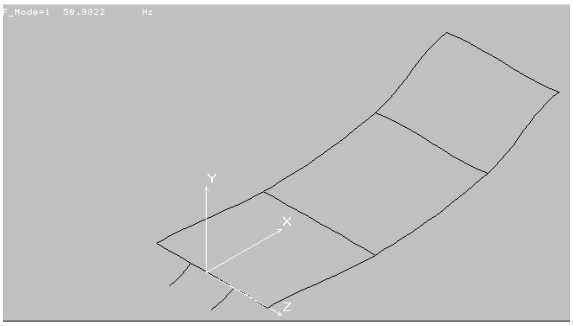


Fig. 12 - The structure deformed shape when it vibrating in the vibration fundamental mode

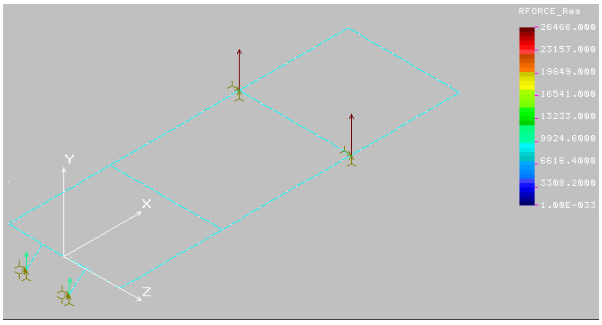


Fig. 13 - The reactions in the bearing points, 26465.7 N on each of the bearing points on the axle and by 11053.9 N on each of the two bearing points from hitch

The mass of the analyzed chassis model is 148.6 kg, the mass center in coordinate system given in Fig. 4 has x_G=1.376 m, y_G=-0.003545 m, z_G= 0.00 m coordinates. All the results above were calculated ignoring gravity loads generated by the own weight of the structure.

If it takes into account the gravitational load, the maximum relative displacement value becomes 1.9987 mm, and the maximum equivalent tension reaches the value of 95,703 MPa, ie minimal changes, negligible. Due to the model simplifications, a series of connecting and strengthening elements (plates, ears, etc..) located on the chassis, are not represented in the structural model, reason way, the value of its mass is less than the value obtained by weighing the real model.

Chassis structural model with axle and wheels, M5

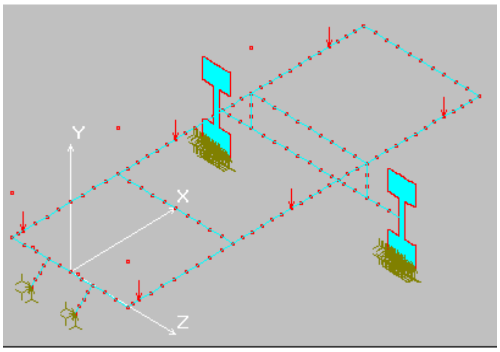


Fig. 14 - The structural model of the chassis with axle, M4

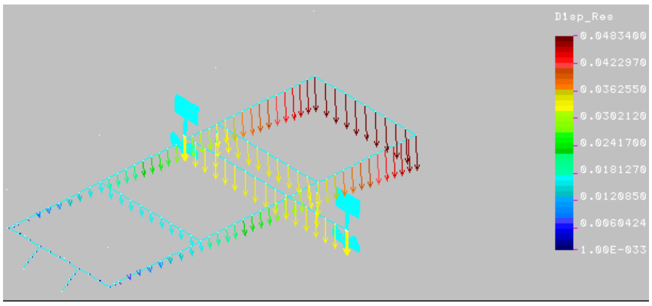


Fig. 15 - The resultant relative displacement distribution in the submodel consists of chassis and axle, as well as in connecting elements between them

This model is obtained by coupling M3 and M4 models. The chassis structural model with axle and wheels is represented in Fig. 14. The bearing is done bottom of wheels and on the chassis front ears. The loading is the same as for the M4 model (by 1250 daN) each loading point marked by red arrows in Fig. 14.

The model response to loads and bearing are given in Fig. 15 and 16. The first ten model fundamental frequencies are: 12,739, 13,459, 29,107, 45,837, 55,267, 66,358, 79,003, 86,442, 129,129, 139,729 Hz.

The M5 model highlights complex issues characteristic of large structures. These results show the effects of considering natural application or closer to reality of the loads. It is changing not only the maximum values of relative displacement of the fields and equivalent tension, but also, the peaks location.

RESULTS AND DISCUSSION

For the M1 model, with data: a = 0.3085 m, l = 1.617 m, F = 37500 N, we obtain the results: M_{max} = 11568.75 Nm, U_{max} = 0.0050 m, respectively maximum tension in the axle, σ = 135.57 MPa, the last has been obtained using Navier's formula, [1] page 317, or [2] page 122, for example, the maximum tension obtained on the exterior fibers of the axle. For the same data above, according with [3], for example, the axle fundamental frequency is 71,759 Hz. The M1 model, also gives the reactions in the bearing points, each equal with F. The model with finite elements, M2, with the same distance between bearings and the same distance between the points of force application, model whose characteristics are given in Fig. 3, has a bearing (border conditions) chosen, so as to obtain results as close to those given by the M1 model, which appears in the literature [1], [2]. The axle arrow (maximum resultant relative displacement) is localized in the middle of the axle, with a value of about 5 mm (Fig. 4). This value coincides with that calculated by conventional formulas given in [2] and is small in relation to axle length between bearings, 1617 mm. The resultant relative displacement, practically coincides with the absolute value of vertical displacement (Oy axis), because loading is static. Maximum equivalent tension is localized in the central area, with the value 135,571 MPa classical calculation by [2] giving practically the same value. The fundamental frequency of vibration is practically the same as that given by the M1 model, 71.76 Hz. The M2 model gives in each of the bearing points a reaction force equal to half of the total load applied to the axle. Reactions can be used to calculate the crushing effort in bearing. The main results of the M2 model and its derivative models appear in Table 1. The M3 structural model is the trailer chassis separat model.

Table 1 - The main results relative only to the axle of the M1 - M5 models

Model	**u_{max}, mm**	**σ_{max}, MPa**	**Fundamental own frequency, Hz**
M1	5.005	135.570	71.759
M2	5.016	135.571	71.762
M21	8.790	219.091	153.544
M22	7.290	214.869	164.251
M23	7.230	215.199	164.731
M3	4.980	134.700	115 – 335
M4	-	-	-
M5	0.700	102.000	143 – 250

CONCLUSIONS AND FUTURE WORK

Due to differences in results provided by the models, two important conclusions detach. The first conclusion is that, physical experiences are the only ones able to validate or to select a most faithful model from a collection of models proposed.

In the absence of experimental data, the reference results for axle modeling are those of the model used for a long time in designing, M1. Such models are used successfully for over a century and gave good results in major European and global industry.

Most of the proposed models give the same deformation of the axle, located in the middle of it, ranging from 5 to 9 mm. However, all these models which give similar values of the deformation in the axle, are loaded with independent forces. A value less than 10 times of the axle deformation indicates the most complex model, M5. This result is due to special loading toward the other models, loading with a force numerically equal, but with the aid of a rigid frame.

Regarding the maximum equivalent tension in the structure, the study main objective - axle – the reference model gives a value which overestimates with about 25% its maximum value. In addition, if the M1 - M3 models locates the maximum value of the equivalent tension to the middle of the axle, while the M5 complex model locates it on the axle in the vecinity of the chassis catching points and in the chassis frame.

It is noted that, the model results [1], although more different than those of the M5 complex model are enough, which explains the proper functioning of the axles in a very long operating time. Reality seems to validate this complex model, [5], [6], [7], [8], [9], [10].

Firmer conclusions in regard to the recommendation of using complex models, that would lead to structures more flexible, lighter, more efficient, we can issue after we have enough data.

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